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Journal of Neolithic Archaeology

www.j-n-a.org

December, 11th, 2013
doi 10.12766/jna.2013.7

Settlement Growth as a Fractal

Aleksandr Diachenko

Abstract

This article deals with a model describing the growth of settlements as a fractal. Long-term settlement growth is a result of non-linear relations between socio-economic development and population size. Our model explains the scaling in settlement populations, the demographic constants, and the conditions for the formation of small centers in the hierarchy of central places.

Introduction

Over the decades, archaeologists have indicated stable population values for human groups all over the world. These stable values or demographic constants within long-term population growth were explained with carrying capacity, economics, social or political organization of the societies and interactions between their members (Birdsell 1953; 1968; Ember 1963; Carneiro 1967; 1987; Reynolds 1972; Glassow 1978; Hassan 1981; Johnson 1982; Kosse 1990; 1994; Dunbar 1992; 1998; 2008; Fletcher 1995; Hill/Dunbar 2003; Kolesnikov 2007; Feinman 2011 et al.; also see critics in Ruiter et al. 2011). This paper shows that there are few equilibrium patterns in the non-equilibrium process of settlement growth. Instead, the population growth of both prehistoric settlements and modern cities are characterized by the same fractal structure. This allows the prediction of population distributions in the future.

Article history:
Received April, 19th, 2013
published December, 11th, 2013

Keywords: Central Place Theory • Fractals • Punctuated Dynamic Equilibrium • Settlement Growth

Cite as: *A. Diachenko: Settlement Growth as a Fractal. Journal of Neolithic Archaeology 15, 2013, 88–105 [doi 10.12766/jna.2013.7].*

Authors' addresse:
Aleksandr Diachenko
Institute of Archaeology of the National Academy of Sciences of the Ukraine
12, Heroiv Stalingrada
Kyiv, Ukraine
04210
oleksandr.diachenko@gmail.com
Tel: +(38096)-547-0619
Fax: +(38044)-418-3306

Authors' Note: This article was written within the framework of the project "The ancient farmers of the Carpathian-Dnieper region: a general model of the socio-economic development" performed within the Fulbright academic exchange program at SUNY at Buffalo, USA in 2010–2011.

Declaration of Conflicting Interests: The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Acknowledgments: I am sincerely grateful to Professor M.J. Woldenberg and Professor E.B.W. Zubrow for the valuable remarks, comments and corrections expressed in the preliminary discussion of the model. I am also grateful to Professor M.J. Woldenberg and Professor E.B.W. Zubrow for the correction of the English text of the article.

Funding: The author received financial support for the Fulbright research project from the Institute of International Education, USA.

JNA

The contemporary state of the correlation between socio-economic development of a society and its population was discussed by R. Fletcher. He showed that different socio-economic systems of various ethnic groups produce equal-sized settlements (Fletcher 2006). G.M. Feinman (2011) argues that the problem of relationship between size and complexity in human collectives requires a third key parameter, integration or interpersonal connectivity.

Other recent papers deal with the fractal population structure of discrete human groups and nonlinear scaling of the space in use. W.-X. Zhou, D. Sornette, R.A. Hill and R.I.M. Dunbar (2005) found a constant scaling ratio of 3 in group sizes. M.J. Hamilton, B.T. Milne, R.S. Walker, O. Burger and J.H. Brown (2007b) indicated a scaling ratio close to 4 in hunters-gatherer societies. M.J. Hamilton, R.S. Walker, J.H. Brown and co-authors showed nonlinear scaling relations between the area required by an individual and the group size among hunters-gatherers (Hamilton et al. 2007a). They also proposed a model of human ecology that includes population dynamics and spatial scaling. The model shows that cooperation between individuals affects equilibrium population sizes and densities (Hamilton et al. 2009). Analyses of the fractal structure of modern city growth are developed in geography (Batty 2005; 2007). All of these studies introduced the problem of the fractal structure of the long-term growth of settlements.

The following models, some were well-known in economic geography before B. Mandelbrot introduced the term "fractal" in 1975, produce fractal-like regularities as well. These models inculde the gravity model, central place theory (hereinafter CPT), the rank-size rule, the allometric growth model (Nordbeck 1965; 1971; Woldenberg/Berry 1967; Woldenberg 1969; 1973; Arlinghaus 1985; Zubrow, 1985; Goodchild/Mark 1987; Arlinghaus/Arlinghaus 1989; Wong/Fotheringham 1990, pp. 90–92; Laxton/Cavanagh 1995; Appleby 1996; West et al. 1997; Brown et al. 2002; 2005, pp. 61–62; Chen/Zhou 2004; 2006; 2008; Chen 2009a; 2009b; 2011 et al.). Since the models from economic geography have fractal properties, they might be useful for the study of population size scaling. This study examines demographic constants and factors that influence long-term growth of settlements. We will start with the correlation between different spatial models and their utility for the analysis of population growth.

A Model

Assumption 1: Assume two equal populations live in settlements with equal resource zones of a limited size. "Political control" is a variable that is spatially distributed. Within the area of political control, the value of the "political control" variable is greater than zero. Beyond the area of political control the variable equals zero. The "political control" of the settlements does not extend beyond the boundaries of their resource zones.

Development: First, we will consider the resource zones. The boundaries of the resource zones of the settlements may be written using the gravity model (Reilly's law) (Reilly 1931):

$$D_{bj} = \frac{D_{ij}}{1+\frac{\sqrt{P_i}}{\sqrt{P_j}}}, \qquad (1)$$

where P_i and P_j are the size of populations for settlements i and j. D_{ji} is the distance between them. D_{bj} is the distance from the settlement j to the boundary of its resource zone at point b, where the resource zone of settlement j touches the resource zone of settlement i.

Second, we will consider "political control." The boundary of the zone under "political control" of a settlement may be written using the "Xtent" model of C. Renfrew and E. Level:

$$P_j^a = cD_{bj}, \qquad (2)$$

where P_j^a is the population of settlement j raised to power a (0.5). D_{bj} is the distance to the boundary of the zone under "political control" at point b; and c is a scaling factor (Renfrew/Level 1979, pp. 149–150). The scaling factor c reflects the total number and volume of a settlement's functions.

Third, assumption 1 makes it possible to integrate the gravity model and the "Xtent" model into a single equation system:

$$\begin{cases} D_{bj} = \dfrac{D_{ij}}{1 + \frac{\sqrt{P_i}}{\sqrt{P_j}}} & (1) \\[2ex] D_{bj} = \dfrac{\sqrt{P_j}}{c} & (2) \end{cases}$$

Thus:

$$\frac{D_{ij}}{1+\frac{\sqrt{P_i}}{\sqrt{P_j}}} = \frac{\sqrt{P_j}}{c} \text{ or} \qquad (3)$$

$$cD_{ij} = \sqrt{P_i} + \sqrt{P_j}.$$

Since the populations of settlements i and j, and their resource zones are equal, the relation between the settlement population (P), the size of its resource zone and the scaling factor may be written as follows:

$$P = c^2 D_{bj}^2 \qquad (4)$$

In fact, equation 4 repeats equation 2. This means that the gravity model with an exponent of 2 and the "Xtent" model with an exponent of 0.5 produce identical results (Kosso/Kosso 1995).

Let us consider the dependence of population growth on the increase of the scaling factor. According to site catchment analysis, the distance from the settlement to the border of its resource zone cannot be more than 5,000 – 6,000 m (Chisholm 1962; Vita-Finzi/Higgs, 1970; Higgs/Vita-Finzi 1972; Jarman 1972; Jarman et al. 1972; Roper 1979 et al.). This means:

$$P = c^2(5{,}000 \ldots 6{,}000)^2. \qquad (5)$$

The exact values of the scaling factor c are not known for an abstract social medium. Therefore, following equation 5, we have generated the population values P for the values of c between 0.001 and 0.015, with intervals of 0.001. The first five are as follows:

$P = 25\ldots36$ (for $c = 0.001$);
$P = 100\ldots144$ (for $c = 0.002$);
$P = 225\ldots324$ (for $c = 0.003$);
$P = 400\ldots576$ (for $c = 0.004$);
$P = 625\ldots900$ (for $c = 0.005$).

Discussion of the preliminary results

The model population values we obtained are close to the demographic constants that were recently carried out by M.A. Kolesnikov, based on data from societies of Australia, Micronesia and the western part of North America. He analyzed data from 238 groups and subgroups in pre-stratified and early stratified societies. He showed that different population groups strive to limit their numbers to 25, 100, 200 and 500 persons (Kolesnikov 2007).

However, the value of 200 obtained by M.A. Kolesnikov is somewhat lower than our corresponding model value of 225…324. Such a difference may be explained by the values of the scaling factor c. There might be rounding error, because c has only three digits after the decimal point. When the value of c is extended to five places after the decimal point, this allows us to obtain more reliable values of the scaling factor, i.e. $c = 0.00083, 0.00183, 0.00283$, etc. (Table 1). By extending the number of decimal places, the results between the model and empirical observations correspond.

Variables	Values				
c	0.00083	0.00183	0.00283	0.00383	0.00483
P	17–25	84–121	200–288	367–528	583–840

Table 1. The interdependence of the values of the settlement population (P) and the scaling factor from the "Xtent" model (c).

It should be noted that M.A. Kolesnikov's demographic constants are the population limits for hunters-gatherers and early agriculturalists. In particular, Australian hunters-gatherers strived to limit their number to 25, 100 and 500, but some groups exceeded this limit (Birdsell 1953; 1968, pp. 229–234; Wobst 1974; Hamilton et al. 2007b). Probably, settled societies produce populations at the low orders approximately equal to hunters-gatherers group sizes. The population numbers ranging from 800 – 2,000 that may be found in Hill/Dunbar 2003; Zhou et al. 2005; Hamilton et al. 2007b for hunter-gatherers regional populations or linguistic units will be discussed below.

As each settlement's population, political influence and resource zone develop, assumption 1 is changed. According to equation 4, the population may grow ($P_0 = P_0N'$) due to the increase of the scaling factor ($c\uparrow, D_{bj} = K'$), the growth of the resource zone of the settlement ($D_{bj}\uparrow, c = K'$), or under the simultaneous influence of two factors ($D_{bj}\uparrow, c\uparrow$), where P_0 is an initial population, N' is a variable, and K' is a constant. Let us analyze these statements.

The values of 5,000–6,000 m for a radius of the resource zone, used in equation (5), were proposed as a limit, never exceeded by the early agriculturalists (for the recent state of the resource zones problem see: Wilkinson 2005). More detailed studies show resource zones that are much smaller in size (Ammerman 1981; Pelisiak et al. 2006; Flannery 2009 et al.). As noted by S. Milisauskas and J. Kruk (1989, p. 406), populations do not adapt to "average" conditions, but to bad years when food resources are limited. Therefore, the population growth in central places due to the increase of the resource zone, when the scaling factor remains stable, seems to be far from reality. However, centers may increase the territory under political control that results in the capital's population growth. This may be exemplified with the major part of Roman Empire history (Fletcher 2006, 131–132). Resource zones of the satellites may grow in order to obtain surplus, and subsequently affect population growth. Most probably, satellites growth due to an increase of the developed territory under constant economic activities reflects the resource potential of various regions.

The larger proportion of the lowest possible radius of a resource zone is controlled by a lower scaling factor. And as the impact of the scaling factor increases, both population and a resource zone increase as well. Our simulations, given in Table 2 and Figure 1, show that the population growth rates are higher than the rates of the resource zone increase. These results correspond to the recent analysis of hunters-gatherers space use. M. J. Hamilton, B. T. Milne, R. S. Walker and J. H. Brown (2007a) showed that the area required by an individual decreases with the increasing of the population size.

	P = 25	P = 100	P = 200	P = 500
c=0.00083	12048.2	24096.4	34096.4	53855.4
c=0.00183	5464.5	10929	15464.5	24426.2
c=0.00283	3533.6	7067.1	10000	15795.1
c=0.00383	2611	5221.9	7389	11671
c=0.00483	2070.4	4140.8	5859.2	9254.7
c=0.00583	1715.3	3430.5	4854.2	7667.2
c=0.00683	1464.1	2928.3	4143.5	6544.7
c=0.00783	1277.1	2554.3	3614.3	5708.8
c=0.00883	1132.5	2265	3205	5062.3
c=0.00983	1017.3	2034.6	2878.9	4547.3

Table 2. The interdependence of the fixed values of the settlement population (*P*), the lowest possible radius of the resource zone and the scaling factor from "Xtent" model (*c*).

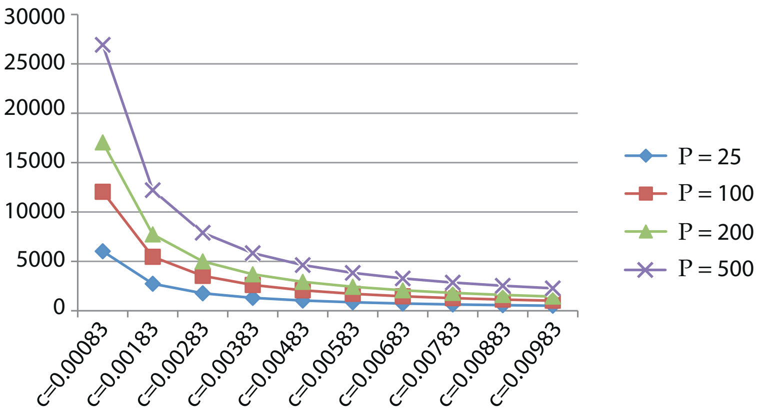

Fig. 1. The interdependence of the fixed values of the settlement population (*P*), lowest possible radius of the resource zone and the scaling factor from the "Xtent" model (*c*).

Perhaps there is a limit to the size of the regions occupied by each group that is determined by transport costs or production efficiency (Algaze, 2008; Woldenberg, pers. comm. on 01.18.2011). Limits to the resource zone require economic and/or social innovations for long-term settlement growth. Since economic and social development involves information exchange and human interactions, the scaling factor c is a key parameter that rules population growth and reflects the interrelations between population size and the resource zones with different properties (Fletcher 1995; Hamilton et al. 2007a; 2009; Feinman 2011).

Strong correlations between the variables in equations (4) and (5) are possible when the population reaches its carrying capacity ($P = P_{max}$). However, populations tend to stabilize below it (Strogatz 2000, Fig. 2.3.3, 21–24). Most probably, stabilization points in settlement growth or the demographic constants (P_s) reflect correlations between the population values and the total number and volume of the settlement's functions. For example, the total population of China increased from 100 million to about 400 million during isolation politics of the Ch'ing dynasty from the XVII – XIX centuries without any effect on the capital's population of about 1 million (Fletcher 2006, p. 131). Such correlations between the demographic constants and the socio-economic development explain the so-called "false urbanization" or population growth that is not supported by economic innovations and social development or a resource zone increase (P_f):

$$P_f > c^2D^2 \qquad (6)$$
$$(P_s < P_f < P_{max};\ c = K';\ D_{bj} = K').$$

Thus, equation (4) is not appropriate and has to be replaced with the following equation set:

$$P_{max} = c^2D^2; \qquad (7)$$

$$P_s = TP_{max}(T < 1); \qquad (8)$$

$$P \neq c^2D_{bj}^2\ (P < P_{max}; P \neq P_s), \qquad (9)$$

where P_{max} is the carrying capacity, P_s is the demographic constant, P is the population below carrying capacity that is not equal to one of the demographic constants. D_{bj} is the radius of the resource zone or area under political control, c is the scaling factor that reflects the number and volume of the settlement's functions, and T is a variable.

Since the simulation synthesis of the gravity model, the "Xtent" model and possible size of the resource zones produce demographic constants, they might be obtained using the equations (7) and (8). These values are described by the Pareto cumulative distribution function with exponent $\alpha = 1$:

$$F_x(x) = 1 - (\frac{x_m}{x})^{\alpha}, \qquad (10)$$

where x is a random variable and x_m is the (necessarily positive) minimum possible value of x.

It should be noted that Pareto distribution is a continuous one. Although the distribution of the population sizes may be described with this function, they remain discrete. If the mode (x_m) is equal to 25, the cumulative distribution function according to M. Kolesnikov's data (2007) takes the values 0.000, 0.750, 0.875, and 0.950.

Several studies have shown that Pareto distribution is appropriate for organizational complexity analysis (Andriani/McKelvey 2009). M. Beckmann's model, which is different from our synthesis above, describes the distribution of cities within the hierarchy of central places. Both result in a distribution described by a Pareto distribution with an exponent of 1 (Beckmann 1958, 245–246). Since our synthesis explains the distribution of cities predicted by CPT, one may conclude that our modeling is an appropriate mode for CPT. Most likely, the use of a gravity model or "Xtent" model in our synthesis determines W. Christaller's "threshold area" and the "range of goods" in central place hierarchies (Parr 1995; Woldenberg, personal communication on 03.09.2011).

The evidence for small centers that have no central functions contradict both the equations 7–9 and the CPT. Moreover, the problem of the function ratio of the settlement and its population number seems to be the most important issue of the CPT application to archeological materials (Adams 1974; Crumley 1976; Smith 1976; Johnson 1977, 494–501; Sindbæk 2007; Nakoinz 2009). W. Christaller notes the existence of hamlets without central functions. Some small agricultural settlements have no economic service functions, but they may serve as social centers or fulfill defense purposes (Christaller 1966, 16, 139–151). The following section examines the factors that influence the formation of the small centers without central functions.

Assumption 2: As an alternative to Assumption 1, suppose that a settlement system is characterized by an intensive growth of population. The settlements fill the region of inhabitance and their distribution is described by M. Beckmann's model. The peculiarities of the economic activity limit the least serviced places to populations of 25 persons and the place with the most services to a population of 500.

Development: M. Beckmann's symbolic model is as follows:

$$p_m = \frac{ks^{m-1}r}{(1-k)^m}, \qquad (11)$$

where p_m is the population of the city of order m, r is the rural population in the market area of the town at order 1, and k is the ratio of the city size to the rural and town population served. The variable s is the parameter of system optimization (Beckmann 1958, 243–244).

Following M. Beckmann, B. Berry and P. Haggett, the index s corresponds to the K-numbers of W. Christaller (Berry 1967, 74–75; Beckmann/McPherson 1970, 25; Haggett 1979, 373).

Solving the problem of the urban and the rural population ratio, equation (12) is derived from equation (11):

$$\frac{(1-k)^m}{k} = \frac{rs^{m-1}}{p_m}. \qquad (11)$$

Since r and p_m are given by Assumption 2 and the values of s correspond to the K-Values K=2, K=3, K=4, and K=7, we have three knowns and two unknowns. Therefore, we can calculate the values for the right part of the equation for different values of m. This provides four knowns and allows us to calculate k. The data is rounded to the nearest thousandth and is given in tables (Table 3). Through the shares of the servicing populations, we obtain the number of inhabitants for each of the elements of the four settlement system structures. This data is rounded to integers and is presented in Table 4.

Table 3. The population share of the centers in the settlement systems with different character of optimization (K) and varied number of the orders in spatial hierarchy (m) after equation (12).

a. K = 2 = s

m	$\frac{rs^{m-1}}{p_m}$	k
3	0.2	0.527
4	0.4	0.377
5	0.8	0.266
6	1.6	0.184

b. K = 3 = s

m	$\frac{rs^{m-1}}{p_m}$	k
3	0.45	0.424
4	1.35	0.243
5	4.05	0.126
6	•	•

c. K = 3 = s

m	$\frac{rs^{m-1}}{p_m}$	k
3	0.8	0.348
4	3.2	0.157
5	12.8	•

d. K = 7 = s

m	$\frac{rs^{m-1}}{p_m}$	k
3	2.45	0.237
4	17.15	•
5	•	•

Table 4. The population of the centers in the settlement systems with different character of optimization (K) and varied number of the order in spatial hierarchy (m).

a. K = 2

Order in hierarchy	Population for a system of m orders				
	Total number of orders				
	2	3	4	5	6
1	25	25	25	25	25
2	500	118	49	25	14
3		500	156	67	34
4			500	183	83
5				500	203
6					500

b. K = 3

Order in hierarchy	Population for a system of m orders			
	Total number of orders			
	2	3	4	5
1	25	25	25	25
2	500	96	32	12
3		500	126	42
4			500	146
5				500

c. K = 4

Order in hierarchy	Population for a system of m orders		
	Total number of orders		
	2	3	4
1	25	25	25
2	500	82	22
3		500	105
4			500

d. K = 7

Order in hierarchy	Population for a system of m orders		
	Total number of orders		
	2	3	4
1	25	25	•
2	500	96	•
3		500	•
4			•

Preliminary results

For settlement systems with K-numbers K=2, K=3 and K=4, the model shows the existence of small centers with the population equal to or less than the population of basic rural settlements (Table 4: a–c). According to the simulations, if we have:

1) a high level of geographical structuring of the settlements;
2) steady values for the population of the largest central place and the basic rural communities over time,

then the small centers may have a population equal to or less than the basic rural settlement.

Next, we will consider the problem of growth for the largest settlements. As noted by D. Pumain, the models of spatial systems, especially when mathematically expressed, describe these systems well enough for relatively static conditions over a time period, but are not adequate to describe long-term historic changes (Pumain 2000, 73–77). Equally, it can be pointed to M. Beckmann's symbolic model, the rank-size rule and allometric growth. The models are extremely similar mathematically (Beckmann 1958, 245–248; Woldenberg/Berry 1967, 131–136; Woldenberg 1973; Batty 2005, 161–162). However, recent analyses show the fractal patterns of human population structure characterized with constant scaling ratios (Zhou et al. 2005; Hamilton et al. 2007b). This allows the generating of an algebraic fractal that includes the demographic constants. The results may be tested with contemporary settlement systems. (It should be noted that B. Mandelbrot (1983, 5) described the etymological opposition of the terms "fractal" and "algebra." However, we combined these terms, as Russian physicists do, for example). The following section will examine the existence of fractal scaling in the distribution of modern cities.

Assumption 3: Let us assume that the size of the population previously found for the service centers becomes a characteristic feature for the serviced places. By applying the iteration of the demographic constants, center growth will be described by the Pareto distribution with an exponent of 1, with the domain values of the cumulative distribution function 0.000, 0.750, 0.875, and 0.950.

Let us assume that the repeating points (repetition of values which occur two times and more in Table 5) generated by the mentioned distribution are an indicator of positive feedback or qualitative changes in economic and/or social subsystems. Positive feedback describes following structural change to the system. A change in element A causes a change in element B which promotes a subsequent change in A away from its original value (Woldenberg/Berry 1967, 129–130; Flannery 1968, 79–81; Renfrew 1972, 23–26; Sherratt 1972, 481–482 et al.). The growth of the settlements is stabilized near the repeating points. Therefore, in this paper the term "repeating point" is equal to "stabilization point" or "demographic constant".

Table 5. The population of city hierarchies with different initial values.

Initial values	**25**	**100**	**200**	**500**	**800**	**2,000**	**4,000**
Values at successive orders	100	400	800	2,000	3,200	8,000	16,000
	200	800	1,600	4,000	6,400	16,000	32,000
	500	2,000	4,000	10,000	16,000	40,000	80,000
Initial values	**16,000**	**64,000**	**128,000**	**320,000**	**512,000**	**1,280,000**	**2,560,000**
Values at successive orders	64,000	256,000	512,000	1,280,000	2,048,000	5,120,000	10,240,000
	128,000	512,000	1,024,000	2,560,000	4,096,000	10,240,000	20,480,000
	320,000	1,280,000	2,560,000	6,400,000	10,240,000	25,600,000	51,200,000
Initial values	**10,240,000**	•	•	•	•	•	•
Values at successive orders	40,960,000	•	•	•	•	•	•
	81,920,000	•	•	•	•	•	•
	204,800,000	•	•	•	•	•	•

Development: The results of applying the Pareto distribution with an exponent of 1, and the domain values of the cumulative distribution function 0.000, 0.750, 0.875, and 0.950 are combined into Table 5. Using the following progression, we can separate four groups of the values (given in thousands):

$$p_m = p_1 K^{m-1}, \qquad (13)$$

where m is the order (0, 1, 2 … n) of the settlement in the spatial hierarchy, K = 2, and p is equal to these values:

- 0.1; 0.2; 0.4; 0.8; 1.6; 3.2; 6.4;
- 2; 4; 8; 16; 32; 64; 128; 256;512; 1,024; 2,048; 4,096;
- 40; 80;
- 1,280; 2,560.

It should be noted that identical progressions are empirically known for the number of service functions of central places in Iowa and in the Aberdeen region of South Dakota (Woldenberg/Berry 1967, 134; Berry 1967, 38). However, this regularity does not refer to four values of population – 0.5, 10,000; 320,000 and 6,400,000 (Table 5).

The regularities, obtained with equation (12), are close enough to the results that were received by M.J. Hamilton, B.T. Milne, R.S. Walker, O. Burger and J.H. Brown (2007b). Since the scaling ratio close to 4 may be written as 2 raised to power 2, the regularity in the structure of hunters-gatherer social networks may be written as follows:

$$p_m \approx p_1(2)^b, \qquad (14)$$

where b is equal to these values: 2; 4; 6; 8 and 10.

Thus, equation (13) covers all the values close to digits that were produced with equation (14).

Now one should return to the repeating points, mentioned in Assumption 3. They are combined into Table 6. In the table one finds the population of the "normal polis" and the proto-polis of antiquity. The former is situated within a limit of 2000–4000 persons and the latter within a limit of 500 persons (Bintliff 2006). The model values also correspond to the values found in inhabited areas in Europe from the Middle Ages until the present (Hodgett 1972; Cherry 1972 20–25, 215–217; Chandler/Fox 1974 et al.).

Table 6. The repeating numbers in Table 5 (These values are the values which occur two times and more in Table 5).

•	10,240,000	•
512,000	1,280,000	2,560,000
64,000	128,000	320,000
•	16,000	•
800	2,000	4,000
100	200	500
•	25*	•

*25 is the basic initiator

Our fractal is divided into three parts beginning with the values 10,240,000; 16,000 and 25 (the basic initiator). Each part begins with a single value that is repeated three times. The first and second parts of the fractal are complete. The third part of the fractal is "growing" (Tables 5 and 6).

The algebraic similarity coefficient of the obtained data is equal to 1/640. One first takes the initial value (25) in the first part of the fractal and divides it by the first initial value (16000) in the second part of the fractal directly below it, whereby the result is 1/640. This relationship is true for each part of the fractal to the right of the first part. In addition, if one takes the next value in the first part, which is 100, and divides it by the next value in the part below it, one also obtains 1/640. The results are similar for each column of numbers to the right and for each row below (see Table 5 and 6: 64,000 x 1/640 = 100; 10,240,000 x 1/640 = 16,000, etc.). Most of the fractals are characterized by the fractional Hausdorff dimension (Mandelbrot 1983, 15). In this case, the Hausdorff dimension (Hausdorff 1919) can be calculated as:

$$D = \frac{\ln Q}{\ln \frac{1}{Z}}, \qquad (15)$$

where D is the Hausdorff dimension; Q is the number of parts for the set and Z is the coefficient of self-similarity.

Therefore, the Hausdorff dimension for this set is:

$$D = \frac{\ln 3}{\ln \frac{1}{1/640}} \approx 0.17. \qquad (16)$$

Direct growth of the settlements is simulated by a generator that is a Pareto distribution function with an exponent of 1 and the values of 0.750, 0.875, and 0.950. In other words, the modal value is increased 4, 8, 20 times correspondingly. It is noteworthy that in the different growth phases repeating points fix different growth rates. In the first phase, the inhabited areas are increased by factors of 4, 8, and 20 times. In other words, growth increases at an accelerating rate. The cycle for the largest centers has a decelerating rate of 20, 8, and 4. In the final phase of the cycle (ratio 0.8, 2 and 4 thousand – 16 thousand), population is also increasing at the same decelerating rate (Table 6). The growth of the population of the settlements is schematically presented in Figure 2. The time periods are indicated on the abscissa as uniform. However, they are not uniform in reality. The growth rate values on the ordinate axis are obtained by means of the addition of the growth rates in each of the phases to the values already achieved. These growth values between 20 and 72 can be somewhat varied, but it does not change the picture of the growth trends. These graphic results are extremely close to the results of E. Zubrow's model, which describes the population growth of a micro-region and its number of settlements. This model includes variables describing the demographic structure of the population (including migration), the duration of settlement functioning, and resource growth (Zubrow 1975 99–111). E. Zubrow's model of prehistoric carrying capacity and the results presented in Table 6 do not contradict the empirically fixed exponential or logistic growth (Haggett 1979, 322–324). They both describe the logistic curve as a single, relatively short-term case in a long-term process. Economic innovations and/or social development increase carrying capacity and related the demographic constant (equations 7–9). Thus, the logistic curve transforms in the more complex graph, consisting of logistic curves that grow one from another (Batty 2007, Fig. 1.2, 27–29).

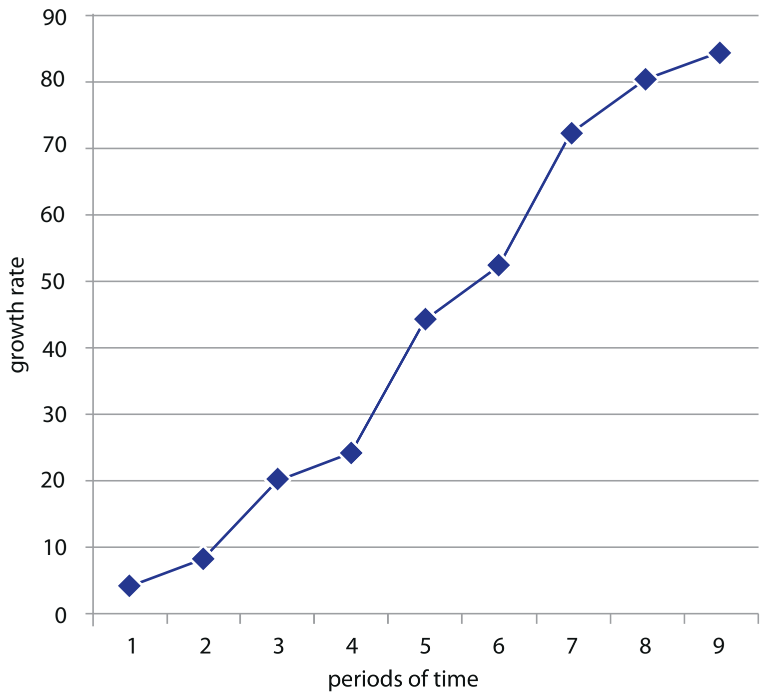

Figure 2. The population growth in the long-term perspective.

In the fractal, the generator of the structure of settled systems is the 0 value of the Pareto distribution function. The modal value is increased one time. In any new level, the generator "reproduces" the population characteristics of the settlements of all previous levels. With this modeling, one can explain the iteration of the population of the inhabited areas with different economic and political systems, which is empirically verified (Bairoch 1988, 3–210, 310–314; Fletcher 2006). In addition, the points generated with the values of the Pareto distribution function of 0.750, 0.875 and 0.950 on each new level are also reproduced. In this case, they provide a diagnosis of positive feedback. Therefore, the growth of the settlements is stabilized near all of those points. An empirical illustration is the distribution of the inhabited areas in Iowa at the beginning of the 1960s. There is a settlement hierarchy in which the number of inhabitants was distributed across settlements that have populations of 100; 500; 1,500; 6,000; 60,000; 250,000 and above 1,000,000 (Berry 1967, 21).

Testing of the preliminary data. The following hypothesis is deduced from the concept relating to growth stabilization in set points. The number of city groups in an actual growing system of settlements classified into orders corresponds with the number of "ideal cities" or settlements of different population size obtained by the model. If such groups include several "ideal cities" (stabilization points), their number corresponds with the number of subgroups, sub-subgroups, sub-sub-subgroups, etc. If the cities belong to groups and subgroups that do not contain "ideal cities", the growth of the population is not steady. It can grow rapidly or collapse rapidly. Sturges' rule and ranking are applied for classification. Sturges' rule is written as:

$$g = 1 + \log_2 n, \qquad h = \frac{(x_{max} - x_{min})}{g}, \tag{17}$$

where x_{max} and x_{min} are the extreme members of the series; g is the recommended interval number; h is the recommended interval length; and n is the total number of data values.

To verify the model, we used the data from "micropolitan" and "metropolitan" areas in the US in 1990 and 2000 (Census 2003, Tab. 1a). We did not divide groups containing the cities with the lower population values (subgroups A and 1, sub-subgroups B, 1 and 2, a) into sub-sub-subgroups because other subgroups are representative enough to verify the model.

The cities in 1990 may be divided into four groups. Their populations range between the following:

I. 12,463 – 1,542,789;
II. 1,542,789 – 4,603,440;
III. 4,603,440 – 12,255,069;
IV. 15,315,720 – 16,846,046.

Group I was divided into six subgroups: A, B, C, D, E and F. Subgroups B and C correspond to the model values 320,000 and 512,000. Subgroups D and E correspond to the model values 1,024,000 and 1,280,000. Subgroup F does not correspond to the model values. Groups II and III were divided into subgroups G, H, I, J, K, L, M, N and O. Subgroups H, I, L, M and O correspond to the model values 2,048,000; 2,560,000; 4,096,000; 5,120,000 and 10,240,000. The model value of 6,400,000 does not correspond to any empirical values. The maximum value of 16,846,046 included into group IV also does not correspond to model data (Table 7).

Table 7. US Cities in 1990 and the Model Values.

Groups and subgroups	Intervals	Number of cities	Model values
I	12,463–1,542,789		
A	12,463–150,809	703	
B	150,809–427,502	127	
B, 1	150,809–218,954	56	
B, 2	218,954–286,910	26	256,000
B, 3	286,910–388,844	39	320,000
B, 4	388,844–422,822	6	-
C	427,502–704,195	35	512,000
D	704,195–1,119,235	21	1,024,000
E	1,119,235–1,395,928	7	1,280,000
F	1,395,928–1,534,274	9	-
II	1,542,789–4,603,440		
G	1,636,528–1,844,917	3	-
H	2,067,959–2,238,480	3	2,048,000
I	2,382,172–2,588,793	7	2,560,000
G	3,069,425	1	-
K	3,686,592–3,767,335	2	-
L	3,989,294–4,248,699	5	4,096,000
III	4,603,440–12,255,069		
M	5,435,468	1	5,120,000
-	-	-	6,400,000
N	8,182,076	1	-
O	11,273,076	1	10,240,000
IV	16,846,046	1	-

The cities in 2000 may be divided into four groups. Their populations are between the following:

I. 13,004 – 1,677,549;
II. 1,677,549 – 6,671,185;
III. 8,335,730 – 13,329,366;
IV. 16,658,457 – 18,323,002.

Group I was divided into five subgroups: 1, 2, 3, 4, 5, 6, 7 and 8. Sub-subgroups 2, b and 2, c of subgroup 2 correspond to the model values of 256,000 and 320,000. Subgroup 3 corresponds to the model value of 512,000. Subgroups 5 and 6 correspond to the model values of 1,024,000 and 1,280,000. Subgroups 4, 7 and 8 do not correspond to model values. Group II was divided into subgroups 9, 10, 11, 12 and 13, corresponding to the model values 2,048,000; 2,560,000; 4,096,000; and 5,120,000. During ten years, the population of the Philadelphia metropolitan area (5,687,147) came closer to the model value of 6,400,000. Two values included into group III correspond to the model value of 10,240,000. The population of the New York metropolitan area came closer to the model value of 20,048,000 (Table 8).

Table 8. US Cities in 2000 and the Model Values.

Groups and subgroups	Intervals	Number of cities	Model values
I	13,004-1,677,549		
1	13,004–161,327	691	
2	161,327–457,974	133	
2,a	161,327–198,112	38	
2,b	198,112–269,431	35	256,000
2,c	269,431–447,728	60	320,000
3	457,974–606,297	21	512,000
4	606,297–902,944	27	-
5	902,944–1,051,268	2	1,024,000
6	1,051,268–1,347,914	14	1,280,000
7	1,347,914–1,496,238	1	-
8	1,496,238–1,644,561	7	-
II	1,677,549–6,671,185		
9	1,735,819–2,179,240	8	2,048,000
10	2,395,997–3,254,821	9	2,560,000
11	4,123,740–4,452,557	4	4,096,000
12	4,175,407–5,165,544	4	5,120,000
13	5,687,147	1	6,400,000
III	8,335,730–13,329,366	2	10,240,000
IV	18,323,002	1	20,480,000

The negative case of the model predicts a rapid growth of 13.5% of the US cities over 218,954 since 1990 and 18.13% of the US cities over 198,112 since 2000. These results are confirmed by the population growth rates for 86.96% of the cities that should not have steady population growth since 1990. 26.08% of these cities had a very rapid growth rate of 21.3 – 38.4% (Census 2003, Table 1; Mackun 2005). According to the model, the population of the New York metropolitan area should stabilize near the point of 20,048,000. Cities that had a population of 606,297 – 902,944 and 1,347,914 – 1,644,561 in 2000 should grow very rapidly or decrease in population.

Discussion of the preliminary results. The model is efficient to analyze the growth of the population aggregates during long time periods and, probably, also medium term perspectives. The model values reflect positive feedback in the system transformation. Demographic constants indicate the optimal balance in population values, economics and socio-political organization and the spatial location of the population.

Population growth is described with an algebraic fractal with Hausdorff dimension approximately equal to 0.17. The model is used to describe population growth in positive feedback, with an increase in economic activity and political influence of the population aggregates, and the so-called “false urbanization”.

Conclusions and Discussion

In the long-term perspective, settlement growth is the dynamic process in a punctuated equilibrium state. Nonlinear relations between the population number, the economic and the socio-political organization produce relatively long periods of stasis, interrupted by rapid bursts (Rosenberg 1994; Bak 1996; Lyman/O’Brien 1998; Gould /Eldredge 2003; Bentley 2003; Ramsey 2003; Chatters/Prentiss 2005; Zeder 2009; Dow/

Reed 2010). According to the model equations (7–9), population stabilizes near discrete fixed points corresponding to the number and volume of a settlement's functions. As soon as population grows above these points, getting closer to carrying capacity, population pressure affects both economy and social organization. This results in fluctuations in settlement growth. Settlements grow to the next stabilization point or decline. Population pressure increases a probability of economic innovations and socio-political transformations.

The long-term growth of settlements is described with an algebraic fractal with a Hausdorff dimension of 0.17, set by the discrete fixed points or demographic constants. It should be noted that M. J. Woldenberg (1968; 1969) also found fixed points in the number of complementary regions. The model generator is synthesized from the gravity model, the "Xtent" model and the site catchment analysis. This confirms the utility of these models, with some modifications, for the study of the long-term settlement growth. The generator is close to M. Beckmann's model, describing the city distribution in the hierarchy of the central places. However, the generator is not identical to this distribution.

When describing settlement growth rate, the proposed model is close to the data of E. Zubrow's (1975) model of prehistoric carrying capacity. Both E. Zubrow's model and our model describe the logistic curve as a single, relatively short-term case in a long-term process. The proposed model is characterized by a discrete distribution of stabilization points for settlement growth. In this aspect, the model agrees with W. Christaller's Central Place Theory. The scaling ratio produced by a model is approximately equal to the ratio indicated for hunter-gatherer group size; our simulations show trends in land use similar to those found for hunter-gatherers (Hamilton et al. 2007a; 2007b). Since economic and social development involves information exchange and human interactions, a settlement's number and volume of functions is a key parameter that rules the population growth in the long-term perspective and reflects the interrelations between population size and the resource zones with different properties (Fletcher 1995; Hamilton et al. 2007a; 2009; Feinman 2011).

The simulations that develop assumption 2 find the conditions when the small centers in the hierarchy of central places are characterized with the smaller values of the population in comparison with the basic rural settlements. These conditions are: the population growth with a high level of geographical structuring of the settlements and steady values for the population of basic rural settlements and the largest central place.

We hope that this paper will be helpful for the further work on the problems of settlement growth. The work on the structure of settlement systems and the short-term fluctuations in settlement populations seems to be the priority for future development of the proposed model. Another important aim for future research is also the analysis of the ratio of economic and political development in settlement growth.

References

Adams 1974: R. McC. Adams, Anthropological perspectives on ancient trade. Current Anthropology 15(3), 1974, 239–258.

Algaze 2008: G. Algaze, Ancient Mesopotamia at the dawn of civilization. The evolution of an urban landscape (Chicago/London 2008).

Ammermann 1981: A. J. Ammerman, Surveys and archaeological research. Annual Review of Anthropology 10, 1981, 63–88.

Andriani/Mckelvey 2009: P. Andriani/B. McKelvey, From Gaussian to Paretian thinking: causes and implications of power laws in organizations. Organization Science 20(6), 2009, 1053–1071.

Appleby 1996: S. Appleby, Multifractal characterization of the distribution pattern of the human population. Geographical Analysis 28(2), 1996, 147–160.

Arlinghaus 1985: S.L. Arlinghaus, Fractals take a central place. Geografiska Annaler B 67(2), 1985, 83–88.

Arlinghaus/Arlinghaus 1989: S.L. Arlinghaus /W.C. Arlinghaus, The fractal theory of the central place geometry: a Diophantine analysis of fractal generators for arbitrary Löschian numbers. Geographical Analysis 21(2), 1989, 103–121.

Bairoch 1988: P. Bairoch, Cities and economic development from the dawn of history to the present (Chicago 1988).

Bak 1996: P. Bak, How nature works: the science of self-organized criticality (New York 1996).

Batty 2005: M. Batty, Network Geography: relations, interactions, scaling and spatial processes in GIS. In: P. Fisher / D. J. Unwin (eds.), Re-Presenting GIS (Chichester 2005).

Batty 2007: M. Batty, Cities and complexity. Understanding cities with cellular automata, agent-based models, and fractals (Massachusetts/London, England 2007).

Beckmann 1958: M. J. Beckmann, City hierarchies and the distribution of city size. Economic Development and Culture Change 6 (3), 1958, 243–248.

Beckmann/McPherson 1970: M. J. Beckmann/J. C. McPherson, City size distribution in a central place hierarchy: an alternative approach. Journal of Regional Science 10(1), 1970, 25–33.

Bentley 2003: R.A. Bentley, An introduction to complex systems. In: R.A. Bentley/H. D. G. Maschner (eds.), Complex systems and archaeology (Salt Lake City 2003) 9–23.

Berry 1967: B. J. L. Berry, Geography of market centers and retail distribution (Englewood Cliffs 1967).

Bintliff 2006: J. Bintliff, City-country relationships in the «normal polis». In: R. M. Rosen/I. Sluiter (eds.), City, countryside, and the spatial organization of value in Classical Antiquity (Leiden-Boston 2006) 13–32.

Birdsell 1953: J.B. Birdsell, Some environmental and cultural factors influencing the structuring of Australian aboriginal populations. The American Naturalist 87 (834), 1953, 171–207.

Birdsell 1968: J.B. Birdsell, Some predictions for the Pleistocene based on equilibrium systems among recent hunters-gatherers. In: R.B. Lee / I. DeVore (eds.), Man the hunter (Chicago 1968) 229–240.

Brown et al. 2002: J. H. Brown/V.K. Gupta/B.-L. Li/B. T. Milne/C. Restrepo/ G. B. West, The fractal nature of nature: power laws, complexity and biodiversity. Philosophical Transactions of the Royal Society of London B 357 (1421), 2002, 619–626.

Brown et al. 2005: C. T. Brown/W. R. T. Witschey/L. S. Liebovitch, The broken past: fractals in archaeology. Journal of Archaeological Method and Theory 12 (1), 2005, 37–78.

Carneiro 1967: R. L. Carneiro, On the relation between size of population and complexity of social organization. Southwestern Journal of Anthropology, 23 (3), 234–243.

Carneiro 1987: R. L. Carneiro, The evolution of complexity in human societies and its mathematical expression. International Journal of Comparative Sociology XXVIII(3-4), 1987, 111–128.

Census de jure, US. Table 1a. (2003). Population in metropolitan and micropolitan statistical areas in alphabetical order and percent change for the United States and Puerto Rico.

Chandler/Fox 1974: T. Chandler/G. Fox, 3000 years of urban growth (New York/London 1974).

Chatters/Prentiss 2005: J. C. Chatters/W. C. Prentiss, A Darwinian macro-evolutionary perspective on the development of hunter-gatherer systems in Northwestern America. World Archaeology 37 (1), 2005, 46–65.

Chen 2009a: Y. Chen, Analogies between urban hierarchies and river networks: fractals, symmetry, and self-organized criticality. Chaos, Solitons and Fractals 40 (4), 2009a, 1766–1778.

Chen 2009b: Y. Chen, Urban gravity model based on cross-correlation function and Fourier analyses of spatio-temporal process. Chaos, Solitons and Fractals 41 (2), 2009b, 603–614.
Chen 2011: Y. Chen, Fractal systems of central places based on intermittency of space-filling. Chaos, Solitons and Fractals 44 (8), 2011, 619–632.
Chen/Zhou 2004: Y. Chen/Y. Zhou, Multi-fractal measures of city-size distributions based on the three-parameter Zipf model. Chaos, Solitons and Fractals 22 (4), 2004, 793–805.
Chen/Zhou 2006: Y. Chen/Y. Zhou, Reinterpreting central place networks using ideas from fractals and self-organized criticality. Environment and Planning B 33 (3), 2006, 345–364.
Chen/Zhou 2008: Y. Chen/Y. Zhou, Scaling laws and indications of self-organized criticality in urban systems. Chaos, Solitons and Fractals 35 (1), 2008, 85–98.
Cherry 1972: G.E. Cherry, Urban change and planning. A history of urban development in Britain since 1750 (Henley-on-Thames 1972).
Chisholm 1962: M. Chisholm, Rural settlement and land use: an essay in location (London 1962).
Christaller 1966: W. Christaller, Central places in Southern Germany (Englewood Cliffs 1966).
Crumley 1976: C.L. Crumley, Toward a locational definition of state systems of settlement. American Anthropologist. New series 78 (1), 1976, 59–73.
Dow/Reed 2011: G.K. Dow/C.G. Reed, Stagnation and innovation before agriculture. Journal of Economic Behavior and Organization 77 (3), 2011, 339–350.
Dunbar 1992: R. Dunbar, Why gossip is good for you. New Scientist 1848, 1992, 28–31.
Dunbar 1998: R.I.M. Dunbar, The social brain hypothesis. Evolutionary Anthropology 6 (5), 1998, 178–190.
Dunbar 2008: R.I.M. Dunbar, Cognitive constrains on the structure and dynamics of social networks. Group Dynamics: Theory, Research and Practice 12 (1), 2008, 7–16.
Ember 1963: M. Ember, The relationship between economic and political development in nonindustrialized societies. Ethnology 2(2), 1963, 228–248.
Feinman 2011: G.M. Feinman, Size, complexity and organizational variation: a comparative approach. Cross-Cultural Research 45(1), 2011, 37–58.
Flannery 1968: K.V. Flannery, Archeological systems theory and Early Mesoamerica. In: B.J. Meggers (ed.), Anthropological archeology in the Americas (Washington 1968) 67–87.
Flannery 2009: K.V. Flannery, Empirical determination of site catchments in Oaxaca and Tehuacán. In: K.V. Flannery (ed.), The early Mesoamerican village. Updated edition (Walnut Creek 2009) 103–117.
Fletcher 1995: R. Fletcher, The limits of settlement growth: A theoretical outline (Cambridge 1995).
Fletcher 2006: R. Fletcher, Materiality, space, time, and outcome. In: J. Bintliff (ed.), A companion to archaeology (Padstow 2006) 110–4.
Glassow 1978: M.A. Glassow, The concept of carrying capacity in the study of culture process. Advances in Archaeological Method and Theory 1, 1978, 31–48.
Goodchild/Mark 1987: F.M. Goodchild/D.M. Mark, The fractal nature of geographic phenomena. Annals of the Association of American Geographers 77 (2), 1987, 265–278.
Gould/Eldredge 2003: S.J. Gould/N. Eldredge, Punctuated equilibrium comes of age. Nature 366 (6452), 2003, 223–227.
Hamilton et al. 2007a: M.J. Hamilton/B.T. Milne/R.S. Walker/J.H. Brown, Nonlinear scaling of space use in human hunter-gatherers. Proceedings of the National Academy of Sciences of the USA 104 (11), 2007a, 4765–4769.
Hamilton et al. 2007b: M.J. Hamilton/B.T. Milne/R.S. Walker/O. Burger/J.H. Brown, The complex structure of hunter-gatherer social network. Proceedings of the Royal Society B, 274 (1622), 2195–2202.
Hamilton et al. 2009: M.J. Hamilton/O. Burger/J.P. DeLong/R.S. Walker/M.E. Moses/J.H. Brown, Population stability, cooperation, and the invisibility of the human species. Proceedings of the National Academy of Sciences of the USA 106(30), 2009, 12255–12260.

Haggett 1979: P. Haggett, Geography. A modern synthesis (New York/Hagerstown/Philadelphia/San Francisco-London 1979).

Hassan 1981: F.A. Hassan, Demographic archaeology (New York/London/Toronto/Sydney/San Francisco 1981).

Hausdorff 1919: F. Hausdorff, Dimension und äußeres Maß. Mathematische Annalen 79 (1–2), 1919, 157–179.

Higgs/Vita-Finzi 1972: E.S. Higgs/C. Vita-Finzi, Prehistoric economies, a territorial approach. In: E.S. Higgs (ed.), Papers in economic prehistory (Cambridge 1972) 27–36.

Hill/Dunbar 2003: R.A. Hill/R.I.M. Dunbar, Social network size in humans. Human Nature 14 (1), 2003, 53–72.

Hodgett 1972: G.A.J. Hodgett, A social and economic history of Medieval Europe (London 1972).

Jarmann 1972: M.R. Jarman, A territorial model for archaeology: a behavioral and geographical approach. In: D.L. Clarke (ed.), Models in archaeology (London 1972) 705–733.

Jarman et al. 1972: M.R. Jarman/C. Vita-Finzi/E.S. Higgs, Site catchment analysis in archaeology. In: P.J. Ucko/R. Tringham/G.W. Dimbleby (eds.), Man, settlement and urbanism (London 1972) 61–66.

Johnson 1977: G.A. Johnson, Aspects of regional analysis in archaeology. Annual Review of Anthropology 6, 1977, 479–508.

Johnson 1982: G.A. Johnson, Organizational structure and scalar stress. In: C. Renfrew M.J. Rowlands/B.A. Seqraves (eds.), Theory and explanation in archaeology: the Southampton conference (New York 1982) 389–421.

Kolesnikov 2007: M.A. Kolesnikov, «Magic numbers» in demography of the pre-stratified and stratified societies. In: M. Videiko/S. Kot (eds.), The Tripolye culture: searches, discoveries and the world context (Kyiv 2007) 79–81 (in Russian).

Kosse 1990: K. Kosse, Group-size and societal complexity: thresholds in the long-term memory. Journal of Anthropological Archaeology 9 (3), 1990, 275–303.

Kosse 1994: K. Kosse, The evolution of large complex groups: a hypothesis. Journal of Anthropological Archaeology 13 (1), 1994, 35–50.

Kosso/Kosso 1995: P. Kosso/C. Kosso, Central Place Theory and the reciprocity between theory and evidence. Philosophy of Science 62 (4), 1995, 581–598.

Laxton/Cavanagh 1995: R.R. Laxton/W.G. Cavanagh, The rank-size dimension and the history of site structure for survey data. Journal of Quantitative Anthropology 5 (4), 1995, 327–358.

Lyman/O'Brien 1998: R.L. Lyman/M.J. O'Brien, The goals of evolutionary archaeology: history and explanation. Current Anthropology 39 (5), 1998, 615–652.

Mackun 2005: P.J. Mackun, Population change in metropolitan and micropolitan statistical areas: 1990–2003. Population estimates and projections. In: US Census Bureau. Current population reports, 2005, 25–1134.

Mandelbrot 1983: B.B. Mandelbrot, The fractal geometry of nature (New York 1983).

Milisauskas/Kruk 1989: S. Milisauskas/J. Kruk, Neolithic economy in Central Europe. Journal of World Prehistory 3 (4), 1989, 403–446.

Nakoinz 2009: O. Nakoinz, Studies on central places and Central Place Theory. Archaologisches Korrespondenzblatt 39 (3), 2009, 361–380 (in German).

Nordbeck 1965: S. Nordbeck, The law of allometric growth. In: J.D. Nystuen (ed.), Michigan inter-university community of mathematical geographers, Paper Nr. 7, University of Michigan, Department of Geography (Ann Arbor 1965).

Nordbeck 1971: S. Nordbeck, Urban allometric growth. Geografiska Annaler B 53 (1), 1971, 54–67.

Parr 1995: J.B. Parr, Alternative approaches to market-area structure in the urban system. Urban Studies 32 (8), 1995, 1317–1329.

Pelisiak et al. 2006: A. Pelisiak/M. Rybicka/M. Ralska-Jasiewiczowa, From the Mesolithic to Modern Times. Settlement organization and economy recorded in annually laminated sediments of the lake Gościąż (Rzeszów 2006).

Pumain 2000: D. Pumain, Settlement systems in the evolution. Geografiska Annaler B 82 (2), 2000, 73–87.

Ramsey 2003: C.B. Ramsey, Punctuated dynamic equilibria: a model for chronological analysis. In: R.A. Bentley/H.D.G. Maschner (eds.), Complex systems in archaeology (Salt Lake City 2003) 85–92.
Reilly 1931: W.J. Reilly, The law of retail gravitation (New York 1931).
Renfrew 1972: C. Renfrew, The emergence of civilization: the Cyclades and the Aegean in the third millennium BC (London/New York 1972).
Renfrew/ Level 1979: C. Renfrew/ E.V. Level, Exploring dominance: predicting polities from centers. In: C. Renfrew/K.L. Cooke (eds.), Transformations. Mathematical approach to cultural change (New York/San Francisco/ London 1979) 145–167.
Reynolds 1972: V. Reynolds, Ethology of urban life. In: P.J. Ucko/R. Tringham/G.W. Dimbleby (eds.), Man, settlement and urbanism (London 1972) 401–408.
Roper 1979: D.C. Roper, The method and theory of site catchment analysis. Advances in Archaeological Method and Theory 2, 1979, 119–140.
Rosenberg 1994: M. Rosenberg, Pattern, process in hierarchy in the evolution of culture. Journal of Anthropological Archaeology 13 (4), 1994, 307–340.
Ruiter et al. 2011: J. Ruiter/G. Weston/S.M. Lyon, Dunbar's number: group size and brain physiology in humans reexamined. American Anthropologist 113 (4), 2011, 557–568.
Sherratt 1972: A.G. Sherratt, Socio-economic and demographic models for the Neolithic and Bronze Ages of Europe. In: D.L. Clarke (ed.), Models in archaeology (London 1972) 477–542.
Sindbæk 2007: S.M. Sindbæk, Networks and nodal points: the emergence of towns in early Viking Age Scandinavia. Antiquity 81 (311), 2007, 119–132.
Smith 1976: C.A. Smith, Causes and consequences of central-place types in Western Guatemala. In: C.A. Smith (ed.), Regional analysis, Vol. 1: Economic systems (London 1976) 255–294.
Strogatz 2000: S.H. Strogatz, Nonlinear dynamics and chaos with applications to physics, biology, chemistry, and engineering (Cambridge, MA 2000).
Vita-Finzi/Higgs 1970: C. Vita-Finzi/E. Higgs, Prehistoric economy of the Mount Carmel Area of Palestine: site catchment analysis. Proceedings of Prehistoric Society 36, 1970, 1–37.
West et al. 1997: G.B. West/J.H. Brown/B.J. Enquist, A general model for the origin of allometric scaling laws in biology. Science 276 (5309), 1997, 122–126.
Wilkinson 2005: T.J. Wilkinson, Approaches to modeling archaeological site territories in the Near East. In: C.S. Beekman/W.W. Baden (eds.), Nonlinear models for archaeology and anthropology. Continuing the revolution (Aldershot, England/Burlington, Vermont 2005) 126–138.
Wobst 1974: H.M. Wobst, Boundary conditions for Paleolithic social systems: a simulation approach. American Antiquity 39 (2), 1974, 147–178.
Woldenberg 1968: M.J. Woldenberg, Energy flow and spatial order. Mixed hexagonal hierarchies of central places. The Geographical Review 58 (4), 1968, 552–574.
Woldenberg 1969: M.J. Woldenberg, Spatial order in fluvial systems: Horton's laws derived from mixed hexagonal hierarchies of drainage basin areas. Geological Society of America Bulletin 80(1), 1969, 97–111.
Woldenberg 1973: M.J. Woldenberg, An allometric analysis of urban land use in the United States. Ekistics 215, 1973, 282–290.
Woldenberg/Berry 1967: M.J. Woldenberg/B.J.L. Berry, Rivers and central places: analogous systems? Journal of Regional Science 7 (2), 1967, 129–139.
Wong/Fotheringham 1990: D.W. Wong/A.S. Fotheringham, Urban systems as examples of bounded chaos: exploring the relationship between fractal dimension, rank-size and rural to urban migration. Geografiska Annaler B 72 (2/3), 1990, 89–99.
Zeder 2009: M.A. Zeder, The Neolithic macro-(R)evolution: macroevolutionary theory and the study of culture change. Journal of Archaeological Research 17 (1), 2009, 1–63.
Zhou et al. 2005: W.-X. Zhou/D. Sornette/R.A. Hill/R.I.M. Dunbar, Discrete hierarchical organization of social group sizes. Proceedings of the Royal Society B 272 (1561), 2005, 439–444.
Zubrow 1975: E.B.W. Zubrow, Prehistoric carrying capacity. A model (Menlo Park, California 1975).
Zubrow 1985: E.B.W. Zubrow, Fractals, cultural behavior, and prehistory. American Archaeology 5(1), 1985, 63–77.

Imprint

ISSN 2197-649X

Layout draft: Holger Dieterich, Ines Reese
Section editor: Martin Hinz
Technical edition: Ines Reese
Language revision: Eileen Küçükkaraca

Published by the Institute of Prehistoric and Protohistoric Archaeology, CAU Kiel;
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